THE LEAP MINUTE—PREDICTING THE UNPREDICTABLE

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Amidst the controversial effort to eliminate leap seconds from Coordinated Universal Time (UTC), the so-called *leap minute* has been recurrently tendered as an alternative approach to reconcile atomic time with astronomical time-of-day. This paper discourses some civil-timekeeping requirements addressed by intercalary minutes, factors that could affect leap-minute scheduling, and the supposed advantages and disadvantages of such a compromise proposal. Particularly, the leap minute does not appear to have obvious and overwhelming advantages over the convention it proposes to replace; furthermore, the inaugural leap minute is expected to happen beyond the professional lifetimes of current advocates, and there is no evidence that its official adoption now would ensure its operational acceptance later.

INTRODUCTION

Within the context of a contentious debate to redefine Coordinated Universal Time (UTC), a wide range of spectators have suggested the so-called *leap minute* as an alternative method for reconciling atomic time with Earth rotation. Apparent supporters include a former director of the International Bureau of Weights and Measures (BIPM), specialists in the timekeeping industry, unnamed officials associated with the Radiocommunication Sector of International Telecommunication Union (ITU-R), expert consumers of civil time, interested journalists, technology bloggers, and some members of the general public. Popular press reports have further echoed the promotion of leap minutes by those closely associated with the process that led to a call for more studies by the 2012 ITU-R Radiocommunication Assembly.^{1, 2} A subsequent Bloomberg editorial expressed the situation this way:[†]

"Several years ago, some scientists suggested scheduling a leap hour for the year 2600. This idea was abandoned as impractical, given that the instructions would have to be left for people six centuries hence. But could there instead be, say, a leap minute every half century?"

PERSPECTIVES REGARDING THE LEAP MINUTE

Observant Citizenry

The leap minute—a clock adjustment introduced when its difference from UT1 approach onehalf minute— is often perceived as a practical compromise by followers of the debate over UTC redefinition. Examples of supportive commentary (with added highlighting) go like this:

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[†] http://www.bloomberg.com/news/2012-02-29/time-is-of-the-essence-in-deciding-fate-of-the-leap-second-view.html

- "...a **30-second discrepancy** between the clocks and the astronomical noon wouldn't hurt anyone."*
- "The average person would not notice if sunrise is off by 30 seconds."[†]
- "Maybe we should wait 100 years and then have a leap minute."[‡]
- "...I wonder why we don't use **leap minutes** rather than leap seconds. The Earth's elliptical orbit already causes the Sun to appear to move up to 15 minutes slower or faster than mean solar time. An additional variance of one minute from true mean solar time would not be a problem for the average person, and correcting clocks by one minute once or a couple of times a century would be much easier for the engineers to keep track of than these continual one-second corrections. And a leap minute would be much bigger news than a leap second."[§]
- "Alternatively, rather than abandon leap-seconds make it **leap-minutes**. Once in a century we could probably use an extra minute anyway."**

Specialist Consumers

In response to a 2011 questionnaire conducted by the IERS Earth Orientation Center, the notion of leap minutes was mentioned a dozen times by those both favoring and opposing that notion.³ Comments (with added highlighting) included:

- "Leap minutes or leap hours would be very disruptive."
- "Perhaps, a 'leap minute' once a century might do. That would be better than this silly idea of a 'leap hour'."
- "Why not introducing leap minutes instead of leap seconds?" [as an alternative proposal]
- "I am wondering there has been enough discussion regarding introducing 'leap minute' instead of leap second." [as an alternative proposal]
- "But if we want follow day and night variation, then within decades we'll need a **leap minute** or within millennia a leap hour... Are these any better than the leap seconds?"
- "Alternatively, the concept would remain for DUT1 but change only when added up to a 'leap minute'."
- "A more realistic option [than a leap hour] with less undesirable effects would be a 'leap minute', but that would also defer difficult issues irresponsibly."
- "...millennia into the future, it might be more logical to insert a **leap minute**, or better yet, perhaps once a century make accurate clocks that run just a bit slower, thus redefining the length of the second."
- "the small and predictable leap second increments are much more tolerable than larger step adjustments proposed (**leap minute** or leap hour) and less troubling..."
- "I prefer 'leap minute' introduced every 50 or 100 years." [as an alternative proposal]

^{*} http://motls.blogspot.com/2012/01/leap-seconds-may-be-abolished-in-2015.html

[†] http://www.realfreemarket.org/blog/2012/01/22/leap-second-should-be-leap-minute/

^{*} http://phys.org/news/2012-06-added.html

[§] Barreiro, A., (2012), "Why not leap minutes?" Comment on the article "A Glitch in Time."

http://www.skyandtelescope.com/community/skyblog/newsblog/A-Glitch-in-Time-160824935.html

^{*} http://mm.icann.org/pipermail/tz/1999-December/010734.html

- "...the issue is a problem that should not be left for future generations (leap minutes, for example)."
- "A leap minute could be introduced..." [as an alternative proposal]

Of 21 responses that favored another preference besides either the *status quo* or "UTC without leap seconds", four (4) specifically proposed leap minutes as an alternative proposal.

Experts and Officials

The insertion of a leap minute into UTC "in about fifty years" was notably advanced by experts in 2001 on the supposition that it would be "relatively easy to adopt."⁴ However, even before the first leap second in 1972, the leap minute had already been contemplated as a potential adjustment mechanism for civil time based on atomic frequency:⁵

Everyday users would not need to be concerned about the introduction of an occasionally modified, atomic scale of time. Various local universal times, standardized by convention, such as Mountain Standard Time (MST), differ from GMT by an integral number of hours, depending on locations. Such local time scales are often adjusted periodically by one-hour steps to yield "daylight savings time," or to return to "standard" time. Similar local times could be derived just as well from an atomic scale, by similar adjustments and zoning, and by the deletion, every few years, of a small number of "one-second leaps" or, alternatively, in about 50 years, a "one-minute leap."

Much more recently, a co-organizer of the 2011 Discussion Meeting "UTC for the 21st Century" tendered the leap minute as a promising compromise.⁶ At this discussion meeting, attendees were politely challenged by the Right Honourable David Willetts, Member of Parliament and the Minister for Universities and Science, to discover an alternative to the current definition of UTC that could still preserve a link between civil timekeeping and Earth rotation. Although the leap minute was mentioned during discussions, it failed to gain technical acceptance and thus no record of its mention survived the organizers' account of the meeting.^{7,8} An attendee from National Institute of Standards and Technology had already advised that "a minute is an intolerably long period of time. The only advantage is that it pushes the problem so far into the future that no one is worried about it."⁹ Another attendee from the UK National Physical Laboratory acknowledged "A century down the line, we'll need to introduce a 'leap minute', and nobody has any sensible arguments for why that won't be a worse issue" compared to leap seconds.¹⁰

An Explicit Leap-Minute Proposal

A detailed leap-minute proposal was submitted in response to the 2011 IERS questionnaire by Pere Planesas of the Observatorio Astronómico Nacional, in Madrid, Spain.¹¹ The proposal included these specific elements:

- The application of a leap minute should target a year when predicted (UTC-UT1) equals 60 seconds.
- The time of insertion should prefer June 30th, because this date less is disruptive than New Year's Eve.
- The announcement of a leap minute should be made "several years" ahead, "strictly" keeping observed (UTC-UT1) between 55.0 s and 65.5 s on the date of application.

No disadvantages from leap minutes were acknowledged in the proposal, but some advantages of a leap minute were noted:

• Leap minutes keep UTC "close to" mean solar time, maintaining UTC's name and status.

- Δ UT1 corrections would be used more to recover UT1, giving more visibility to those who determine these corrections.
- The first leap minute would not occur for several decades, allowing all hardware, software, and time-dissemination standards to adapt to the change of convention.
- Fewer adjustments would be required per century, allowing the difference between International Atomic Time (TAI) and UTC to remain constant for decades.
- Leap minutes cope well with quadratic UT1 separation in the very long-term, and also avoid the possibility of negative corrections.

Although Planesas' leap-minute proposal is detailed, the requirements addressed by its specifications are unclear. The introduction of a leap minute when (UTC–UT1) is ~60 s maximizes the magnitude of (UTC–UT1) and pushes the intercalary minute as far as possible into the future. The prescription that a leap minute must occur within a window of ± 5 s is not seemingly based on specific technical criteria, yet it limits the amount of advanced notice from decades to years. And there is also no technical rationale for the notion that "UT1 plus one minute" is a viable substitute for specifications calling for mean solar time at Greenwich. The proposal presumes that an intercalary minute could only occur at either the end of June 30th or December 31st UTC, though leap seconds can be introduced at the end of any Gregorian calendar month.¹² And, while leap minutes might cope with quadratic UT1 separation better than, say, leap seconds, that technical aspect may not be relevant for perhaps a millennium.

THE TIMING OF A LEAP MINUTE

The question of when to introduce a leap minute is mostly an inquiry about the future separation of Universal Time (UT1) from Terrestrial Time (TT)^{*}, also known as ΔT :

$$\Delta T = TT - UT1 \tag{1}$$

Forecasts of ΔT are sometimes used for planning the potential magnitude and frequency of intercalary clock adjustments, but such forecasts are notoriously inaccurate. For example, Essen (1967) suggested that such differences should vary less than 20 seconds over the next two centuries:¹³

"Consider now the consequences of using the atomic time scale. The atomic unit has been made as nearly as possible equal to the second of Ephemeris Time (ET). During the 200 years or so during which ET has been compared with UT, the maximum deviation of the two scales amounted to $\pm 20s$. If the earth continues to behave in a similar way, and the atomic scale is set to give the time of day as given by astronomical measurements, then the time it records should not differ from UT by more than $\pm 20s$ during the next 200 years."

In hindsight, Essen's prediction greatly underestimated the rate of separation, for it took less than one-quarter century, rather than two centuries, for the difference between UT and atomic time to deviate 20 seconds. More recently, Vincent Meens, chairman of the ITU-R Study Group responsible for recommending a redefinition of UTC, reportedly said "The [rate of] deviation of the leap

^{*} The predecessor of TT was Ephemeris Time (ET).

second is about 1 minute and 30 seconds per century. The deviation for a millennium will be on the order of 15 minutes."*

The predictive statements of Essen and Meens seem to presume that ΔT increases linearly. In contrast, and within the context of redefining UTC, Nelson *et al.* (2001)⁴ illustrated a parabolic separation, consistent with a change in the excess length of day of 2.3 ms/cy (or ΔT change of 42 s/cy²). Such rapid separation is compatible with a theory of lunar tides by which tidal braking results in the observed tidal acceleration of the Moon, without other torques or modeled changes to the principal moment of the inertia of the Earth.¹⁴ However, this degree of change is not observed in ΔT since the invention of the telescope, and significantly overstates the observed retardation of UT1 in recent years. The early work of Clemence (1948) estimated a value much less: the average deceleration in Earth rotation "over the past 2000 years is 29 seconds per century per century" (a change in the excess length of day of 1.6 ms/cy), predicting that "[b]y A.D. 3000 the accumulated effect will amount to something like half an hour."¹⁵

Parabolic Approximation of ΔT

Parabolic representation of ΔT , often used for general trend analyses and extrapolation, assumes constant deceleration of Earth rotation, or, deceleration that varies somewhat randomly around a single mean value. This is equivalent to saying that the rate of change in the excess length of day (LoD), a representation of the time derivative of ΔT , is asymptotically constant. However, parabolic fits highly depend on the data or theory adopted. For a parabola mathematically expressed as:

$$\Delta T_{\text{modeled}} = a + c \left(t - t_0 \right)^2, \qquad (2)$$

the most influential information regarding the shape parameter c is available when $(t - t_0)$ is large. However, over the telescopic era for which ΔT has been observed precisely, $(t - t_0)$ is comparatively small.

Phenomena not modeled by a constant rotational deceleration cause inaccurate extrapolations. Long-term changes to the total angular momentum or mass moment of inertia of the figure of the Earth, and/or changes in the differential rates between internal parts of the Earth (e.g., core v. mantle), complicate the supposed parabolic shape of ΔT . The local change-point in the behavior of ΔT at the end of the 20th century, and its subsequent dearth of leap seconds, is a recent reminder that the rotation rate of Earth has an unpredictably random component. For ancient observations before the invention of the telescope, decadal variations in the terrestrial rate of rotation introduce irregular fluctuations about the mean trend of ΔT , leading to inherent geophysical uncertainty in ancient observations of at least ± 20 seconds.¹⁶ Some authors have used mathematical analysis of the historic ΔT series to suggest the possible existence of harmonics signatures in the excess length of day, which has led to some speculation about the possibility of a future negative leap second.¹⁷ Stephenson & Morrison (1995), while hypothesizing the existence of very longperiod harmonics, also recognize significant differences in near-term and long-term estimates of length-of-day.¹⁸ Within statistical accuracy, Huber (2006) modeled the stochastic behavior of LoD process with three components (a global Brownian motion process, decadal fluctuation, and a 50-day oscillation), and hypothesized that the component of acceleration often attributed to post-glacial rebound might also be explainable, in principle, as a purely random affect.¹⁹

^{*} http://www.abc.net.au/science/articles/2012/01/18/3410293.htm

Thus, parabolic approximation of ΔT is a relatively naïve approach to predicting the rotation of Earth, even though this model is often employed for long-term extrapolation. Various parabolic estimates illustrate the significant uncertainty of a parabolic approach by producing a wide range of future values that could predict the insertion of a leap minute into UTC (Figure 1).

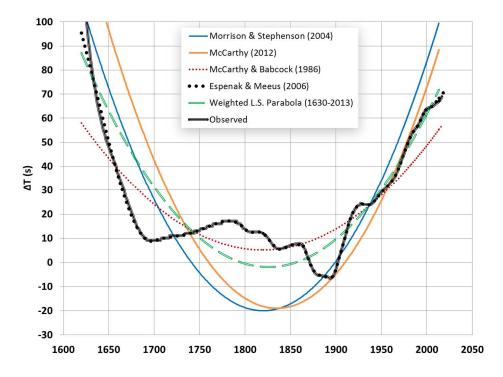


Figure 1. Observed ΔT With Parabolic Approximations.

Very Long-Term Approximations of ΔT

*Morrison & Stephenson (2004).*²⁰ Stephenson, Morrison, and their colleagues have proposed various long-term parabolic approximations for estimated ΔT series spanning several thousand years, their work leveraging chronicled observations of ancient eclipses. For the tidal deceleration of the Moon, their research culminated into the general recommendation that researchers use:

$$\Delta T_{\text{M\&S}(2004)} = -20 + 32 \left[(t - 1820.0)/100 \right]^2 \text{s}, \tag{3}$$

for extrapolating beyond the limits of ΔT tables, assuming a lunar mean-motion rate of $\dot{n} = 26''/cy^2$.

*McCarthy (2012).*²¹ McCarthy explored the introduction of future intercalary seconds on a divergent^{*} yet predictable schedule. Consideration of a graphical illustration of the parabolic curve attributed to "Mathews & Lambert" suggests that the model closely approximates:

^{* &}quot;Divergent" means the proposed schedule does not constrain |UT1–UTC| to any specific value, as is the current practice, but tracks the extrapolation of a "parabolic fit based on geophysics" over the next five centuries.

$$\Delta T_{\rm McC(2012)} = -19 + 33.8 \left[(t - 1835.0)/100 \right]^2 s, \tag{4}$$

The value of 33.8 s/cy² in Eq. (4) is adapted from an estimate of 1.85 ms/cy from Mathews and Lambert (2009) for the total rate of change in length of day.²² Mathews and Lambert assign a standard error of ± 1 ms/cy to the rate estimate.

Approximations of ΔT over the Telescopic Era

*Espenak & Meeus (2006).*²³ Espenak & Meeus (2006) discuss the uncertainty in Earth's rotational period and its impact on the geographic visibility of eclipses in the past and future, using higher-order polynomials over specific time intervals to approximate the behavior of ΔT . Their extrapolation of ΔT from 2005 to 2050 is:

$$\Delta T_{\text{E\&M}(2006)} = 62.92 + 0.32217(t - 2000.0) + 0.005589(t - 2000.0)^2 \,\text{s},\tag{5}$$

This expression is derived from a fit to predictions of ΔT in 2010 and 2050. The estimate at 2010 (66.9 s) was based on a linear extrapolation from 2005 using 39 s/cy (the average rate from 1995 to 2005). The estimate at 2050 (93 s) was linearly extrapolated from 2010 using 66 s/cy (the average rate from 1901 to 2000).

*McCarthy & Babcock (1986).*²⁴ McCarthy & Babcock provide a useful reduction of ΔT and length-of-day estimates based on observational data after 1656, including error estimates, assuming a lunar deceleration rate of $\dot{n} = 26$ "/cy. The following parabolic fit was determined:

$$\Delta T_{\text{McC}\&B(1985)} = -5.156 + 13.3066 \left[(t - 1819.0)/100 \right]^2 \text{ s}, \tag{6}$$

with an uncertainty of $\pm 0.3264 \text{ s/cy}^2$ assigned to the quadratic term. An analysis by this author determined that Eq. (6) is an unweighted least-squares curve fit to the semi-annual ΔT series from 1657.0 to 1984.5.

Weighted Least-Squares Parabola. The near-term predictive performance of Eq. (6) can be made more accurate by weighting ΔT according to the estimated error of its series. A weighted least-squares estimate using the same ΔT series and errors estimates from McCarthy & Babcock (1986) over 1657.0-1984.5 yields:

$$\Delta T_{\text{Weighted (1657-1985)}} = -1.73 + 20.8 \left[(t - 1826.5)/100 \right]^2 \text{ s} . \tag{7}$$

The value of the quadratic term of Eq. (7) falls well outside the uncertainty assigned by McCarthy & Babcock to the same term from Eq. (6), but remains within the uncertainty assigned by Mathews and Lambert (2009) to the quadratic term of Eq. (4).

If the observational series of McCarthy & Babcock (1986) is extended forward up to 2013.0, and backward to 1630.0, a weighted solution becomes:

$$\Delta T_{\text{Weighted (1630-2013)}} = -1.92 + 21.0 \left[(t - 1825.8)/100 \right]^2 \text{ s}, \qquad (8)$$

For Eq. (8), semi-annual ΔT values from 1985.0 to 2013.0 were taken from USNO Series 7 tables and assigned an uncertainty of 1 ms. From 1630-1655, ΔT values at five-year increments were used from Stephenson & Morrison (1984)^{*} and assigned an uncertainty of 20 s per Morrison & Stephenson (2004).^{20, 25} The inclusion of six additional weighted data points from 1630-1655

^{*} This series is also available from Section K of the Astronomical Almanac.

changed curve by ~ 0.3 s at 1650, and made a negligible change near the year 2000. Over the fitinterval of several centuries, graphs of Eq. (7) and Eq. (8) appear practically identical. For the purposes of this work, Eq. (8) is adopted and represented in all figures, this estimate having considered the most observational data.

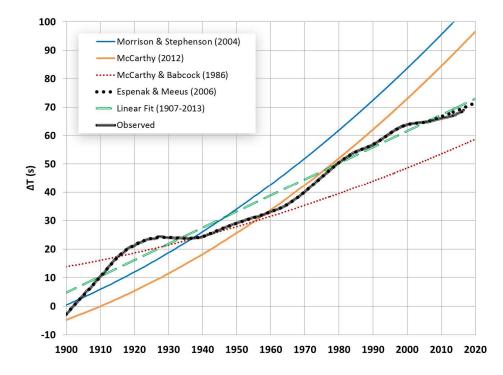


Figure 2. ΔT during the 20th Century. A linear fit to ΔT over 1907-2013 is included, from which TT-UT1 changed by one minute.

Degenerate Parabolæ. The historical sequence of ΔT back to 1907—over which ΔT changed by one minute—does not generally follow the long-term parabolic trends ascribed by researchers to lengthier historical series. Indeed, a linear fit to ΔT over the 20th century might seem just as reasonable as any other low-order empirical approximation relative to the data available. The coarse representation of ΔT as a sequence of degenerate parabolæ, or piecewise linear approximations, each spanning decades to centuries, seems viable in light of compelling work by Stephenson *et al.* (1997), which suggests, for example, a nearly linear trend in ΔT from mid-16th century to the late 17th century. This trend accommodates a historically important record of a rare type of solar eclipse^{*} observed by Christoph Clavius in 1567, during a time where ΔT is otherwise poorly established before the availability of telescopic observations.²⁶ Other piecewise linear approximations might be imagined also spanning the 18th and 19th centuries, and again spanning the 20th century. A linear fit implies that Earth's mean rotation rate stays approximately constant over the time interval to which it is fit. The advantage of piecewise approximation is that it does not attempt to assign one deceleration value throughout history like a single parabola.

^{*} The apparent diameters of the Moon and Sun were so well matched that the eclipse was neither total nor annular, rather, sunlight remained passing through lunar valleys.

An unweighted linear trend, fit to semi-annual ΔT values from 1907.0 to 2013.0, results in:

$$\Delta T_{\text{Linear}(1907-2013)} = -11.0 + 55.2 \left[(t - 2000.0)/100 \right] \text{s} .$$
(9)

The disadvantage of a naïve linear fit is that its predictability is questionable, because Earth's rotation can dramatically and permanently depart from the local tendency at any time. Nevertheless, it seems useful to consider such an estimate to provide yet another trend for establishing a wider perspective on the problem of ΔT predictability.

Leap-Minute Prediction by Extrapolation

The redefinition of UTC is a proposed agenda item for the ITU-R World Radio Conference (WRC) of 2015. Draft recommendations have already proposed that UTC redefinition occur five (5) years after adoption by the WRC; thus, A.D. 2020 is used here as a starting point for analysis. The USNO currently estimates that ΔT will have reached 70 s at this starting date;^{*} such that if leap minutes become operational, then UTC with leap minutes (UTC_{LM}) is presumed to revert to an atomic scale equal to approximately (TT – 70) s at the time of adoption.

If ΔT tracks one of the trends in Figure 1, then Table 1 suggests the times at which each UT1 trend will have separated from (TT – 70) s. The estimated times are provided for separation values of 30 s, 60 s, and 90 s after 2020. (That is to say, these are dates at which extrapolations of ΔT curves in Figure 1 and Figure 2 reach 100 s, 130 s, and 160 s.) The dates for $\Delta = 30$ s suggest a very wide range of potential dates for leap-minute insertion.

Extrapolation	Year $\Delta = 30s$	Year $\Delta = 60s$	Year $\Delta = 90s$
Extrapolation	$\Delta = 508$	$\Delta = 008$	$\Delta = 908$
Morrison & Stephenson (2004)	(2010)	2036	2057
McCarthy (2012)	2022	2045	2064
Weighted Parabola Fit (1630-2013)	2046	2076	2103
Espenak & Meeus (2006)	2057	2084	2106
Linear Fit (1907-2013)	2067	2120	2173
McCarthy & Babcock (1986)	2086	2125	2160

Table 1. Dates at which (TT since 2020 - 70 s) separate from various ΔT trend approximations.

Another predictive approach might assume that UT1 will simply increase according to the long-term trend at the time of changeover from leap seconds to leap minutes. See, for example, the work of Steve Allen[†], and McCarthy & Klepczynski (1999).²⁷ This approach is illustrated in Figure 3 by simply shifting various parabolic trends vertically to align them with the expected value of ΔT at 2020, and then extrapolating the parabolic trend. The advantage of this approach is that it forces all predictions to begin from the same origin of adoption for comparative purposes. However, if a particular representation of the long-term trend of ΔT is asymptotically correct, then the shifted prediction will be biased relative to the asymptotic behavior.

^{*} ftp://maia.usno.navy.mil/ser7/deltat.preds

[†] http://www.ucolick.org/~sla/leapsecs/dutc.html

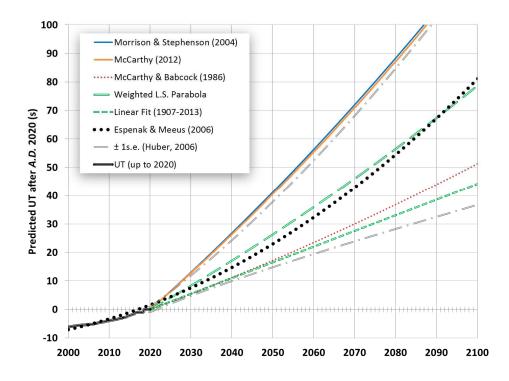


Figure 3. Forecasts Based on Mean-Trend Accelerations. Post-2013 UT based on USNO forecast.

The dating results from Figure 3 are provided in Table 2. Added to Figure 3 are long-term standard-error bounds due to Huber (2006), which are primarily based on an assumed global Brownian motion process. These standard-error bounds are drawn relative to the weighted least-squares parabola. (This curve was chosen because it is generally central to all the other curves.) Although the Huber standard-error bounds cannot be considered an exact guide, they serve to illustrate that such extrapolations, which graphically appear very different, may be statistically no different from each other to within reasonable confidence levels. That is to say, various differences in long-term prediction may have less to do with changes in the long-term rate of acceleration, and more to do with estimation uncertainty caused by random fluctuations in fitted ΔT . According to Table 2 then, one may conclude that a wide range of possible dates could be legitimately assigned to first leap minute.

Table 2. Dates since 2020 at which UT1 separates from (TT-7	0) s
from various accelerating UT1 Forecasts.	

Extrapolated Curve	Excess LoD Growth (ms/cy)	Year $\Delta = 30s$	Year $\Delta = 60s$	Year $\Delta = 90s$
Morrison & Stephenson (2004)	1.75	2042	2062	2081
McCarthy (2012)	1.85	2042	2063	2081
Weighted Parabolic Fit (1630-2013)	1.15	2053	2083	2109
Espenak & Meeus (2006)	3.06	2057	2084	2106
McCarthy & Babcock (1986)	0.72	2069	2111	2147
Linear Fit (1907-2013)	0.00	2072	2125	(2178)

Designating the Time of Century

Because of the wide range of possible dates at which a leap minute might occur, it becomes practically impossible to assign far into the future the optimal date which would guarantee, say, $|UT1-UTC_{LM}|$ to be "thirty-something" seconds. Thus, the adoption of a small threshold for $|UT1-UTC_{LM}|$ would preclude assignment of a specific date for the first leap minute at the time the new scheme is adopted. Such a scenario negates much of the presumed benefit of the proposal and raises questions as to the scheme's future success, because a supposed criterion to establish the date of the first adjustment could be changed (or abolished) after the fact. The recourse would adopt an insertion date based on some *presumed* behavior for UT1 far in advance. Minimizing $|UT1-UTC_{LM}|$ by introducing a leap minute at \sim 30 s suggests an insertion date generally near the middle of the 21st century, according to Table 2.

Designating the Time of Year

Another factor that comes into consideration is the time of year at which an adjustment should occur. Electronic traffic and commerce will almost certainly be disrupted by the fact that a leap minute would need to maintain an atypical representation such as 23:60:--, just as leap seconds are labeled 23:59:60. A date should therefore be chosen to minimize its impact on commerce. This suggests that a leap minute should occur on the equivalent of an international holiday.*

The Gregorian system, or Western calendar, is used extensively today a *de facto* international standard.^{28, 29} Consequently, the first day of the Gregorian calendar, January 1, is now observed almost universally as a public holiday.³⁰ Experience suggests that some technicians find various time-representation glitches caused by end-of-year roll-over, so increased awareness of potential problems may already exist at this time of year more than any other. Also, recent history has shown that many people expect leap seconds to occur at the end of the year, perhaps because the end-of-year adjustments receive more media attention around the New Year's Eve holiday. There might be a strong sociological argument for maintaining intercalary adjustments at the end of the year, considering that New Year's Eve is mainly a celebration of the passage of time.

The disadvantage or inconvenience of choosing a holiday is that it requires more experienced personnel monitoring potential technical issues that may arise from such a rare event, during a time that traditionally would be spent away from regular duties. Also, more and more commerce is being practiced on holidays, especially electronically, and thus the technical advantages of a holiday date may wane with time. An alternative choice might associate the leap minute with February 29^{th} , which emphasizes that the leap minute is a long-term correction to the astronomical calendar.

Designating the Time of Week

Another consideration is the day of week on which an adjustment might fall, as there may be some debate as to whether a weekday or a weekend would be preferable. Presuming that western industrial conventions for a Monday-through-Friday work week are still being practiced decades from now, the weekend may be a time when commerce and data collection would be least disrupted by a calendrical adjustment. The final moments of Saturday on the meridian of Greenwich would result in the leap minute occurring on Sunday for eastern half of the globe, and Saturday for the western half. However, many Middle-Eastern countries currently practice a Fri-

^{*} This is in contrast with Planesas' suggestion that the leap-minute insertion date avoid New Year's Eve.

day/Saturday rest period. Also, it is not clear if some cultures would object to the perception of having to supply any extra manpower during a time period that is normally dedicated to rest.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
2032	2044	2028	2040	2024	2036	2020
2060	2072	2056	2068	2052	2064	2048
2088		2084	2096	2080	2092	2076

Table 3. Day of Week for February 29th, 2020-2092

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	2018	2019		2020	2021	2022
2023		2024	2025	2026	2027	
2028	2029	2030	2031		2032	2033
2034	2035		2036	2037	2038	2039
	2040	2041	2042	2043		2044
2045	2046	2047		2048	2049	2050
2051		2052	2053	2054	2055	
2056	2057	2058	2059		2060	2061
2062	2063		2064	2065	2066	2067
	2068	2069	2070	2071		2072
2073	2074	2075		2076	2077	2078
2079		2080	2081	2082	2083	
2084	2085	2086	2087		2088	2089

Table 4. Day of Week for December 31st, 2018-2089

Memorability of the Date

Table 3 and Table 4 tabulate the days of the week during much of the 21st century on which the dates of February 29th and December 31st fall. If primary consideration is given to an insertion date near the middle of the 21st century, and if weekend insertion is also given priority, then the dates of February 29, 2048 and December 31, 2050 appear promising according to a "Satur-day/Sunday weekend" preference, or December 31, 2049/January 1, 2050 according to a "Fri-day/Saturday weekend" preference. However, when scheduling a rare event far into the future, the memorability of the date may also play an important role. More significance may be attached to a mid-century date like December 31, 2049, compared to, say, February 29, 2048. Traditionally, the fundamental time epoch for astronomical theories have often been fixed to half-century increments, such as 1800, 1850, 1900, 1950, and 2000.

THE REPRESENTATION OF A LEAP MINUTE

There are three basic means of reconciling atomic time and astronomical time of day. If the constancy of unit duration is maintained, this results in intercalary adjustments having unconventional representations (such as 23:60). Otherwise, units of duration must be varied or "stretched" to fit traditional representations of time in software and hardware. Outside of stretching or leaping, the only remaining option is to replace one atomic timescale with another once the difference from astronomical time is out of tolerance, much like the Gregorian calendar replaced the Julian calendar. Implementation of a leap minute could be approached with any one of these methods. However, at this stage it is unclear how a proposed leap minute should work any differently than the leap second works now. Specifically, lengthening a UTC "day" by inserting exactly sixty (60) TAI seconds at the end of the UTC day seems to appeal as a default approach, rather than insertion at any other hour of the day.

Eyeing the distant future, it seems unlikely that hardware manufacturers would build specialized circuitry, gearing, infrastructure, or provide other complex arrangements, to account for a leap minute, and there is little expectation that people will discard and replace existing hardware for a once-in-a-lifetime event. Simply taking devices temporarily out of service is likely to be an attractive option, as there are likely to be many computer codes that would otherwise interpret 23:60:-- as 00:00:-- of the next day, *etc*. The duration of a leap minute is significant enough that many relatively inaccurate timekeepers would not be able to ignore it; thus, awkward representational issues with leap minutes would be more widespread than with leap seconds.

Because there is an implicit requirement that the display of a leap minute must comply with historic systems of time representation, it seems reasonable to exclude the possibility of representation as a sequence of conventional leap seconds labeled from 60 to 119, because any representation that requires three-digit representation would be incompatible with digital time-display formats expecting no more than two-digits worth of seconds. Consequently, the interval of a leap minute might be expected to have a digital representation of seconds lasting from zero through 59. Or, representational issues might be implemented as a temporary rate adjustment to local hardware clocks (such as UTC rate slewing of the 1960's, the recent Google "leap smear",^{*} or UTC-SLS[†]).

THE REQUIREMENTS SURROUNDING LEAP MINUTES

The justifications often offered for a proposal are indicative of the inherent requirements to be addressed. Recognizable advantages supposedly addressed by a leap minute have thus far included: public concerns about the separation of clock time from the Sun, operational convenience, and preservation of "Coordinated Universal Time" as a named timekeeping convention. Each of these is now discussed in detail.

Public Concerns

Hudson (1967) noted that, for the leap minute specifically, "approximate epochal coherence with the rising and setting of the sun would be retained, and there need be no fear of a radical departure from solar time for 'everyday' purposes."⁵ Specifically, there is natural variation in the timing of celestial phenomena from year to year due the fact that the beginning of the calendar year shifts with respect to the solar year by as much as ³/₄ of a day at addition of February 29th

^{*} http://googleblog.blogspot.com/2011/09/time-technology-and-leaping-seconds.html

[†] http://www.cl.cam.ac.uk/~mgk25/time/utc-sls/

(Figure 4). The physical radius of the Sun subtends about one-half degree of arc, such that solar timing can also vary by about one minute of time depending on whether one is referencing the limb of the Sun, or its center. The annual timings of sunrise and sunset are only published to a precision of one minute in the *Astronomical Almanac*, and local atmospheric conditions, terrain obstructions, and changes in observer elevation, can add significantly to the unpredictably of apparent rise and set times Thus, leap minutes appear to maintain the association of annual lighting conditions and clock time at a level of precision useful for some "everyday" applications, while also providing a symbolic mechanism to address public concerns that clock time should correlate with Earth rotation.

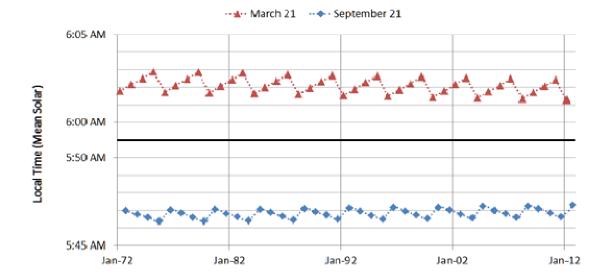


Figure 4. Time of Astronomical Sunrise (McCormick Observatory, Charlottesville, Alt. 264 m M.S.L.). Sunrise is when the center of the solar disc is 50' below the local horizon plane.

However, leap minutes do not offer any advantages over leap seconds with regard to supposed public concerns over harmonization of clock with sky. Already, frequent leap seconds noticeably provide "approximate epochal coherence" to a higher level of accuracy, and for a wider range of technical applications. And because the coupling of civil time and Earth rotation is relaxed with leap minutes, the symbolism is not as strong and thus potentially less satisfying.

Advanced Notice v. Frequency of Adjustment

There is an operational need for intercalary adjustments to be predicted "sufficiently" far in advance. The debatable point is what constitutes "sufficiently". Leap seconds tend to be announced six months in advance, but they could be announced earlier.³¹ UTC_{LM} allows for extremely long-term announcement only if a small tolerance for $|UT1-UTC_{LM}|$ is not expected. Very long-term announcement is not documented to be an operational requirement; instead, "sufficient" frequency of adjustment is required. For example, Kamp (2011) argues that intercalary adjustments "once every couple of years is not nearly often enough" to ensure that systems handle them correctly.³² Huston (2012) echoes a similar opinion as it relates to leap-second bugs: "It is often the case in systems maintenance that the more a bug is exercised the more likely it is that the bug will be isolated and corrected."³³ An isolated leap minute very far in the future therefore seems most inconvenient to the immediate testing requirements of developers, which is in stark contrast to one of the primary academic arguments offered for its support, *i.e.*, "[t]he idea behind

the move to suppress the insertion of leap seconds is that less inconvenience would be caused if, instead, a leap minute or so were inserted perhaps once every century."⁶

"Advanced notice" also implies adequate infrastructure for promoting intercalary adjustments. Intercalary adjustments are best transmitted directly to precise timekeeping devices in the short-term.^{*} Interestingly, many devices used for 'everyday' timekeeping (computers, cellular telephones, *etc.*) are inherently poor clocks; they display time accurately because their internal oscillators are frequently synchronized to time signals over communication networks. Because accurate timekeepers tend to be connected to a networked time source, there is already infrastructure in place to propagate intercalary adjustments electronically and automatically, given the appropriate telecommunication protocols. Thus, the distribution of accurate, semi-frequent intercalary adjustments such as leap seconds is a telecommunication concern foremost. Relatively frequent "conventionally scheduled", or divergent, leap seconds could be announced in advance via the same telecommunication protocols as accurately directed leap seconds are now, but their broad-cast would not be as critical a concern for telecommunication, because their advanced scheduling makes other avenues of distribution possible.

In contrast, it is quite unclear how a leap minute—scheduled decades in advance—would be successfully "broadcast" to a future generation, because the announcement of a leap minute does not appear to be a telecommunication issue. Adjustment mechanisms prescribed 100 or even 400 years into the future can be respected, evidenced by the successful employment of Gregorian calendar reforms. However, a calendar is not a clock, and the algorithmic insertion of leap days is not easily compared to a leap minute. Leap days benefit from regular and frequent recurrence, and from having the customary representation of February 29th. A leap minute is without precedent because of its novelty and peculiar representation—perhaps the calendrical equivalent of proclaiming that a December 32nd should happen in the distant future.

Nomenclature

Another supposed advantage of the leap minute is that it supports continuing use of the term "Coordinated Universal Time" for a scale that otherwise practically diverges from Universal Time. This perspective may be supported from the somewhat abstruse fact that 'Coordinated Universal Time' and 'UTC' originally referred to a broadcast convention which employed smaller steps and slight rate adjustments during the 1960's, yet was replaced by a new convention in 1972 which eliminated rate adjustments in favor of larger steps. Arguably then, the leap minute continues an "existing tendency" to further loosen the tie between precise timekeeping and solar-based time in a scale called 'UTC'.³⁴ However, this perspective neglects the fact that UTC was always purposed to provide Universal Time to a precision of 0.1 s via both definitions: before 1972, this was accomplished primarily by frequency steering, whereas after 1971 it was accomplished with leap seconds plus digital encoding of the DUT1 correction. Leap minutes fail to supply Universal Time to any technically useful level outside of, say, *gnomonics*, and thus alter the character and technical purpose of the civil-broadcast scale. This could lead to confusion as the term "UTC" is repurposed to designate quite different concepts for time.³⁵

Other Explanations Not Directly Supporting Requirements

Some justifications for preferring leap minutes do not completely support that option favorably or logically. Some critical discussion follows of arguments often advocating leap minutes.

^{*} If there is too-little advanced notice, then the announcement may not be distributed broadly enough; if there is toomuch advanced notice, operators or devices risk losing track of a scheduled adjustment.

Unnoticeable differences in the timing of celestial phenomena. A recognized position of advocacy is that deviations by up to a minute in $|UT1-UTC_{LM}|$ will not affect the general public's perception of celestial phenomena. However, clocks already do not attempt to indicate apparent solar time, mainly because the "average person" has no strong need to precisely predict celestial phenomena outside of, for example, participation in specific ritual activities. Nonetheless, the effects of a leap minute will be noticeable to those for which the relationships between celestial phenomena, Earth rotation, and precise clock readings are technically important. Rather than the timing of celestial events such as surrise or sunset, a more general consideration for the public might be how accurately clock readings should indicate the beginning and ending of the mean-solar day, which has legal significance in many jurisdictions.

Greater publicity and/or cause for festivity. A leap minute would be a once-in-a-lifetime excuse for celebration and for heightened public curiosity in timekeeping. However, leap minutes actually provide 59 fewer opportunities to "celebrate" and "publicize" versus the current definition of UTC. And, although there is a saying that "any publicity is good publicity," the publicity surrounding a future leap minute might not be celebrated due to the technical inconveniences it might cause. Many "average people" were greatly inconvenienced either directly or indirectly by the so-called "Y2K"* or "Millennium Bug", which required significant resources to investigate and remedy. Arguably, leap seconds receive favorable publicity—mostly as a curiosity—because they have remained largely inconsequential.

Fewer adjustments ease the accounting. Timekeeping history is full of peculiarities such as irregular months, local calendars, and holidays. Continuing simplification has not proven to be a fundamental requirement, evidenced by changing legislation about daylight-saving/summer time, zone-time jurisdictions, *etc.* The very arguments for breaking the tie between UTC and UT1 are the same arguments that could be used to abolish zone time and daylight-saving time / summer time—simplifications which have no widespread public advocacy. If accounting ease was the ultimate goal of the leap minute, then one should expect the first leap minute to never be announced, because no adjustment makes for the simplest of all accounting. (An account into which no withdrawals or deposits are made is forgotten.) Additionally, leap seconds will still need to be accommodated in applications dealing with past data, with leap minutes complicating the historical accounting. Easy accounting thereby argues *against* the leap minute.

Leap minutes harmless to the "average person". Unfortunately, the "average person" of the distant future is hard to define in the present. Such persons will likely depend on advancing technology that could be ill-equipped for leap-minute adjustments in precise civil timekeeping. It seems reasonable to suppose that leap minutes would be much more disruptive to later generations unaccustomed to global intercalary clock adjustments than leap seconds are to the present generation. It also seems reasonable to suppose that leap minutes will be neglected in many circumstances, even if announced decades in advance. (The "Y2K" or "Millennium Bug" existed despite universal knowledge that the year 2000 would come.) Any further loosening of the degree of coupling between UT1 and UTC fosters a situation whereby realignment of civil time with the rotation of the Earth becomes highly impractical because it fails to motivate system designers to accommodate long-term adjustments.

^{*} "Y2K" (Year 2000) refers to systemic issues caused by the habitual representation of two-digit years in data timestamps during the 20^{th} century.

CONCLUSION

To a casual spectator, the leap minute might seem like an elegant and logical compromise: it is sixty times less frequent than the present convention of leap seconds, it is sixty times smaller in magnitude than the previously proposed leap hour, and it addresses the requirement that civil timekeeping maintain coherence with (mean) solar time. However, documented perspectives indicate that the leap-minute approach has been in contention since the 1960's and lacks consensus. It is unclear what technical applications would be satisfied by a deviation of up to one minute between UT1 and UTC versus some other value, and an optimal insertion point cannot be predicted very far in advance to great accuracy. Thus, the function of the leap minute seems entirely aesthetic, with precision time broadcasts becoming effectively decoupled from Earth rotation.

Although proximity to UT1 cannot be relied upon with a leap minute, other factors could establish an insertion date far in advance. These factors might include the memorability of the insertion date, and supposed times of the year and days of the week which might minimize disruptions. However, any significant change from the *status quo* will introduce some degree of confusion and cause some existing systems to malfunction. A leap minute must still label events in an atypical way, as with leap seconds, except a leap minute will be harder to ignore. Considering that as yet "there is no equipment in the world that could handle a leap minute,"³⁶ there is a real risk that a distant adjustment scheme would not be reliably implemented when the declared time comes because the technicalities are pushed so far into the future that they would not be pragmatically addressed.³⁷ Thus, the infrequency of the leap minute fails to meet a technological requirement to maintain critical awareness necessary to support intercalary adjustments. The current system of timekeeping therefore seems to meet existing and future requirements in ways that a leap minute cannot, such that technologists should not act too hastily to alter or discard it.

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